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Reg. No. : .....

Name : .....

**Third Semester B.Sc. Degree Examination, March 2021**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course — II**

**MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS — I**

**(2018 Admission)**

Time : 3 Hours

Max. Marks : 80

**SECTION I**

All the questions are compulsory. Each question carries 1 mark.

1. State the Inclusion-Exclusion Principle for 3 finite sets  $A$ ,  $B$  and  $C$ .
2. Define a Linear Diophantine Equation in two variables.
3. Define Derivative of a vector-valued function  $r(t)$ .
4. Evaluate the definite integral,  $\int_0^2 2ti + 3t^2 j dt$ .
5. Define Unit Normal Vector.
6. Give the formula for  $a_N$ , the normal component of acceleration for a moving particle in terms of the velocity  $v$  and the acceleration  $a$ .

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7. Find the natural domain of  $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$ .
8. Define an Open set.
9. Define Gradient of a function  $f(x, y, z)$ .
10. State the Extreme value theorem.

## SECTION II

Answer **any eight** questions. Each question carries **2** marks.

11. State and prove the Pigeonhole Principle.
12. Express  $3ABC_{\text{sixteen}}$  in base 10.
13. Find  $(4076, 1024)$ .
14. Prove that two integers  $a$  and  $b$  are relatively prime if and only if  $[a, b] = ab$ .
15. Let  $r(t) = t^2 i + e^t j - (2 \cos \pi t) k$ . Find  $\lim_{t \rightarrow 0} r(t)$ .
16. Define a tangent vector and tangent line through the point  $P$  on the graph of a vector-valued function  $r(t)$ .
17. State the chain rule for differentiation of vector functions.
18. A particle moves through 3-space in such a way that its velocity is  $v(t) = i + t j + t^2 k$ . Find the coordinates of the particle at time  $t$ .
19. Define Level surfaces and find the level surfaces of  $f(x, y, z) = z^2 - x^2 - y^2$ .
20. Define local linear approximation to  $f(x, y)$  at  $(x_0, y_0)$ .
21. Consider the ellipsoid  $x^2 + 4y^2 + z^2 = 18$ . Find an equation of the tangent plane to the ellipsoid at the point  $(1, 2, 1)$ .
22. Explain the steps to find the absolute extrema of a continuous function  $f$  of two variables on a closed and bounded set  $R$ .

### SECTION III

Answer **any six** questions. Each question carries **4** marks.

23. Find the number of integers  $\leq 3000$  and divisible by 3, 5 or 7.
24. Prove that if  $a$  and  $b$  are positive integers, then  $[a, b] = \frac{ab}{(a, b)}$ .
25. Using canonical decomposition, find the LCM of 1050 and 2574.
26. Find parametric equations of the tangent line to the circular helix  $x = \cos t, y = \sin t, z = t$  where  $t = t_0$ , and use that result to find parametric equations for the tangent line at the point where  $t = \pi$ .
27. If  $r_1(t)$  and  $r_2(t)$  are two vector functions of  $t$ , derive the expression for  $\frac{d}{dt}(r_1 \times r_2)$ .
28. Find  $r(t)$  given that  $r'(t) = (3, 2t)$  and  $r(1) = (2, 5)$ .
29. Assuming that polynomials in one variable and cosine function are continuous, show that  $f(x, y) = \cos(3x^3 y^4)$  is continuous everywhere. State the results used for the proof.
30. Differentiability of function  $f(x, y)$  at a point implies continuity at that point. Justify the statement.
31. Suppose that  $w = x^2 + y^2 - z^2$  and  $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$  use appropriate forms of the chain rule to find  $\frac{\partial w}{\partial \rho}$  and  $\frac{\partial w}{\partial \theta}$ .

### SECTION IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and Prove the Fundamental Theorem of Arithmetic.
- (b) Let  $b \geq 2$  be an integer and  $b+1$  integers are randomly selected. Prove that the difference of two of them is divisible by  $b$ .

33. (a) If a circle is parameterised by arc length as

$$r(s) = a \cos\left(\frac{s}{a}\right) i + a \sin\left(\frac{s}{a}\right) j \quad (0 \leq s \leq 2\pi a). \text{ Find } T(s) \text{ and } N(s).$$

- (b) Find the curvature  $k(t)$  for the circular helix  $x = a \cos t, y = a \sin t, z = ct$  where  $a > 0$ .

34. Given  $f(x, y) = \frac{xy}{x^2 + y^2}$ , find the limit of  $f(x, y)$  as  $(x, y) \rightarrow (0, 0)$  along

- (a) The x-axis
- (b) The y-axis
- (c) The line  $y = x$
- (d) The line  $y = -x$
- (e) The parabola  $y = x^2$ .

35. Explain Lagrange multipliers Method and use it to determine the dimensions of a rectangular box, open at the top, having a volume of  $32 \text{ ft}^3$ , and requiring the least amount of material for its construction. Given the surface area being  $S = xy + 2xz + 2yz$ .
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