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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme Under CBCSS

Statistics

Core Course II

ST 1341 – PROBABILITY AND DISTRIBUTION – I

(2019 Admission Regular)

Time : 3 Hours Max. Marks : 80

Use of calculator is permitted.

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define supremum of a set.
2. What do you meant by neighbourhood of set?
3. Write an example for a countable set.
4. Find the limit of a sequence (a_n), where $a_n = \left(\frac{3+2\sqrt{n}}{\sqrt{n}}\right)$.
5. Define monotone sequence.
6. Define convergence of a sequence.
7. Define sample space.
8. State multiplication theorem of probability.
9. Define probability mass function.
10. Write an example for a continuous random variable.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define open set. Write an example for open set.

12. When do you say that a set is bounded? Write an example for bounded set.

13. Prove that the set of all rational numbers is countable.

14. Define Cauchy sequence.

15. Show that the sequence (a_n) where $a_n = \left(\frac{n^2+3n+5}{2n^2+5n+7}\right)$ converges to $\frac{1}{2}$.

16. Define root test.

17. Define oscillating sequence. Write an example.

18. What is the use of Raabes test?

19. Examine whether the series $\sum_{n=1}^{\infty}\frac{1}{n^2 \log n}$ convergent or not.

20. Define conditional probability.

21. Given $P(A) = 0.30$, $P(B) = 0.78$ and $P(A \cap B) = 0.16$. Compute $P(A^c \cap B^c)$ and $P(A \cap B^c)$.

22. If A and B are two independent events then prove that A^c and B^c are independent.

23. If the probability mass function of a random variable X is

    $f(x) = x/6,\ x = 1, 2, 3$, find $P(X \geq 2)$.

24. Define distribution function. Write any two properties of distribution function.

25. Define marginal density function.

26. What is meant by mutual independence of events?

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Prove that the union of an arbitrary family of open set is open.

28. Show that

    (a) every finite set is bounded

    (b) every subset of bounded set is bounded.

29. Examine the convergence of the sequence $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$.

30. Using Cauchy's first theorem on limits prove that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}}\right] = 1.$$

31. Prove that every absolutely convergent series is convergent.

32. Describe Cauchy's condensation test.

33. Write the mathematical and axiomatic definitions of probability.

34. State and prove addition theorem of probability for two events.

35. Prove that pairwise independence does not imply mutual independence.

36. If X is a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2}, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$ find the distribution function of X.

37. X is a random variable with probability density function $f(x) = \begin{cases} 3x^2 e^{-x^3}, & x > 0 \\ 0, & otherwise \end{cases}$ and $Y = X^3$. Derive the probability density function of Y.

38. Joint probability density function of a bivariate random variable (X, Y) is given by $f(x, y) = \begin{cases} c(x^2 + y^2) & 0 < x < 2, 1 < y < 4 \\ 0, & otherwise \end{cases}$. Determine c.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. State and prove Bolzano Weierstrass theorem.

40. Establish Leibnitz test.

41. Let $\{a_n\}$ and $\{b_n\}$ be two sequences with limits a and b respectively. Prove that

    (a) sequence $\{a_n + b_n\}$ converges to $a + b$

    (b) sequence $\{a_n b_n\}$ converges to ab and

    (c) sequence $\left\{\frac{a_n}{b_n}\right\}$ converges to $\frac{a}{b}$ if $b \neq 0$ and $b_n \neq 0$ for all n.

42. State and prove ratio test.

43. (a) State and prove Baye's theorem.

    (b) 60% of the students in a class are males and remaining are females. 50% of the male students and 80% of the female students are football lovers. If the elected class representative is football lover, find the probability that the representative is a female.

44. Joint probability density function of a bivariate random variable (X, Y) is given by $f(x) = \begin{cases} \frac{4(x+y)}{5x^3}, & 1 < x < \infty, 0 < y < 1 \\ 0, & otherwise \end{cases}$. Examine the independence of X and Y and compute $P(0 < Y < 0.5 | X = 2)$.

(2 × 15 = 30 Marks)