(Pages : 6) **K – 2425**

Reg. No. : ..............................

Name : ..................................

**Third Semester B.Sc. Degree Examination, March 2021**

**First Degree Programme Under CBCSS**

**Mathematics**

**Core Course**

**MM 1341 — ELEMENTARY NUMBER THEORY AND CALCULUS – I**

**(2019 Admission Regular)**

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions from **1** to **10**. **Each** question carries **1** mark.

1. State the division algorithm.

2. State the pigeonhole principle.

3. If $r(t) = t^2 i + e^t j - (2\cos \pi t\, k)$, find $r'(t)$.

4. Evaluate the definite integral $\int_0^2 (2ti + 3t^2 j)\,dt$.

5. Write the formula for finding the arc length of a parametric curve $x = x(t)$, $y = y(t)$, $a \le t \le b$.

6. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.

7. Let $f(x, y) = \sqrt{y+1} + \ln(x^2 - y)$. Find $f(e, 0)$.

8. Evaluate $\lim_{(x, y) \to (-1, 2)} \frac{xy}{x^2 + y^2}$.

9. Find $\frac{\partial w}{\partial x}$ if $w = 2ye^z \sin xz$.

10. Define the gradient of the function $f(x, y, z)$.

PART – B

Answer **any eight** questions from 11 to 26. **Each** question carries **2** marks.

11. Let $b$ be an integer $\geq 2$. Suppose $b+1$ integers are randomly selected. Prove that the difference of two of them is divisible by $b$.

12. State the Inclusion Exclusion principle.

13. If $a \mid c$ and $b \mid c$, and $(a, b) = 1$, show that $ab \mid c$.

14. If $p$ is a prime and $p \mid ab$, show that $p \mid a$ or $p \mid b$.

15. Sketch the graph and a radius vector of $r(t) = \cos t\, i + \sin t\, j$, $0 \leq t \leq 2\pi$.

16. Find the arc length of that portion of the circular helix $x = \cos t$, $y = \sin t$, $z = t$ from $t = 0$ to $t = \pi$.

17. Determine whether $r(t) = t^3 i + (3t^2 - 2t) j + 5t^2 k$ is smooth or not.

18. Find the unit tangent vector to the graph of $r(t) = t^2 i + t^3 j$ at the point $t = 2$.

19. Find the instantaneous velocity and speed of a particle that moves in a circular path such that its $x$ and $y$ coordinates at time $t$ are $x = 2\cos t$, and $y = 2\sin t$.

20. Find the curvature of a circle of radius $a$.

21. Evaluate $\lim_{(x,y)\to(0,0)} \frac{-xy}{x^2+y^2}$ along the parabola $y = x^2$.

22. Find $f_x$ and $f_y$ for $y(x, y) = 2x^3y^2 + 2y + 4x$.

23. Show that $f(x, y) = x^2 + y^2$ is differentiable at (0, 0).

24. Using the chain rule find $dz / dt$ if $z = x^2y, x = t^2, y = t^3$.

25. Let $f(x, y) = x^2e^y$. Find the maximum value of a directional derivative at (–2, 0) and find the unit vector in the direction in which the maximum value occurs.

26. Find the equation of the tangent plane to the ellipsoid $x^2 + 4y^2 + z^2 = 18$ at the point (1, 2, 1).

PART – C

Answer **any six** questions from 27 to 38. **Each** question carries **4** marks.

27. Let $a$ and $b$ be any positive integers. Show that the number of positive integers $\leq a$ and divisible by $b$ is $[a / b]$.

28. Find the primes such that the digits in their decimal values alternate between 0's and 1's, beginning with 1 and ending in 1.

29. Let $f_i$ denote the $i$-th Fermat number. Show that $f_0 f_1 \ldots f_{n-1} = f_n - 2$, where $n \geq 1$.

30. Show that there are infinitely many primes of the form $4n + 3$.

31. The graphs of $r_1(t) = (\tan^{-1} t)i + (\sin t)j + t^2k$ and $r_2(t) = (t^2 - t)i + (2t - 2)j + \ln t\, k$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin.

32. Find $T(t)$ and $N(t)$ for the circular helix $x = a\cos t$, $y = a\sin t$, $z = ct$, where $a > 0$.

33. Find the curvature and radius of curvature of $r(t) = 5\cos t\, i + 12\sin t\, j + t\, k$ at $t = \pi / 2$.

34. Find the arc length parametrization of the circular helix $r = \cos t\, i + \sin t\, j + t\, k$ that has reference point $r(0)$ = (1,0, 0) and the same orientation as the given helix.

35. Confirm that the mixed partial derivatives of $f(x,y) = 4x^2 - 8xy^4 + 7y^4 - 3$ are equal.

36. Suppose that $w = \sqrt{x^2 + y^2 + z^2}, x = \cos\theta, y = \sin\theta, z = \tan\theta$. Use the chain rule to find $dw / d\theta$ when $\theta = \pi / 4$. ,

37. Find the directional derivative of $f(x,y) = e^{xy}$ at (–2, 0) in the direction of the unit vector that makes an angle $\pi / 3$ with the positive axis.

38. Find the parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1, 1, 2).

## PART – D

Answer **any two** question from 39 to 44. **Each** question carries **15** marks.

39. (a) State and prove the fundamental theorem of arithmetic.

(b) Show that the linear Diophantine equation (LDE) $ax + by = c$ is solvable if and only if $d / c$, where $d = (a,b)$. Also show that If $x_0, y_0$ is a particular solution of the LDE, then all its solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t$, $y = y_0 - \left(\frac{a}{d}\right)t$ where t is an arbitrary integer.

40. (a) Let $\alpha = \frac{1+\sqrt{5}}{2}$. Show that $\alpha^{n-2} < F_n < \alpha^{n-1}$, where $n \geq 3$ and $F_n$ denotes the $n$-th Fibonacci number.

(b) Show that the number of divisions needed to compute ($a$, $b$) by the eu-clidean algorithm is at most five times the number of decimal digits in $b$, where $a \geq b \geq 2$.

41. (a) The length, width and height of a rectangular box are measured with an error of at most 5% Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box.

(b) Let $L(x,y)$ denote the local linear approximation to $f(x,y) = \sqrt{x^2 + y^2}$ at the point (3, 4). Compare the error in approximating $f(3.04, 3.98)$ by $L(3.04, 3.98)$ with the distance between the points (3, 4) and (3.04,3.98).

42. Suppose that a particle moves through 3-space so that its position vector at time $t$ is $r(t) = t\,i + t^2 j + t^3 k$.

(a) Find the scalar tangential and normal components of acceleration at time $t$.

(b) Find the scalar tangential and normal components of acceleration at time $t = 1$.

(c) Find the vector tangential and normal components of acceleration at time $t = 1$.

(d) Find the curvature at the point where the particle is located at time $t = 1$.

43. (a) Find the absolute maximum and minimum values of $f(x,y)=3xy-6x-3y+7$ on the closed triangular region $R$ with vertices (0, 0), (3,0) and (0,5).

(b) Consider the ellipsoid $x^2+4y^2+z^2=18$.

(i) Find an equation of the tangent plane to the ellipsoid at the point (1, 2, 1).

(ii) Find parametric equations of the line that is normal to the ellipsoid at the point (1, 2, 1).

(iii) Find the acute angle that the tangent plane at the point (1, 2, 1) makes with the *xy*-plane.

44. (a) Locate all relative extrema and saddle points of $f(x,y)=4xy-x^4-y^4$.

(b) Find the points on the sphere $x^2+y^2+z^2=36$ that are closest to and farthest from the point (1, 2, 2).

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