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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Statistics

Core Course – 3

ST 1341 — PROBABILITY AND DISTRIBUTION – I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. When do we say that two events are mutually exclusive?
2. State multiplication theorem of probability.
3. Define probability space.
4. Define probability mass function.
5. Find the constant k such that $f(x) = kx^2$, $0 < x < 3$; 0, otherwise is a probability density function.
6. If P(A) = 0.4, $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.1$ find $P(B)$.
7. Define expectation of a random variable.
8. If Cov(X, Y) = 25 compute Cov($2X + 7$, $Y + 10$).
9. Find $M_{a+bX}(t)$, where a and b are constants.
10. Write an example of a random variable whose moment generating function do not exist.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Define sample space. Write an example.

12. If A and B are two events such that $P(A) = 1/3$, $P(B) = 1/2$ and $P(A \cup B) = 17/24$, compute $P(A \mid B^c)$.

13. Write the axiomatic definition of probability.

14. Examine whether the following function is a distribution function or not.

$$F(x) = \begin{cases} 0, & x < -2 \\ \dfrac{x+2}{4}, & -2 \le x \le 2. \\ 1, & x > 2 \end{cases}$$

15. Probability mass function of random variable X is given below. Find the value of k

x:	0	1	2	3	4	5
$P(X = x)$	1/16	1/16	5/8	1/16	1/8	k

16. If (X, Y) has joint pdf $f(x, y) = \frac{3}{4} - \left(\frac{x+y}{8}\right)$, $0 < x < 2,\ 2 < y < 4$. Determine the marginal density functions of X and Y.

17. Define joint distribution function of a bivariate random variable. Express $P(a < X \le b, c < Y \le d)$ in terms of distribution function.

18. If X and Y are independent, then show that $\rho_{xy} = 0$, where ρ is the correlation coefficient.

19. Let (X, Y) be a bivariate continuous random variable. Show that $E[E(X \mid Y)] = E[X]$.

20. Define conditional expectation and conditional variance.

21. State and prove the additive property of cumulant generating function.

22. Define probability generating function. Establish the relationship between probability generating function and moment generating function.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. A problem in statistics is given to three students *A*, *B* and *C* whose chances of solving it are ½, ¾ and ¼ respectively. What is the probability that the problem will be solved if all of them try independently?

24. State and prove Baye's theorem.

25. Let X be a random variable with probability density function $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & otherwise\end{cases}$. Find the probability mass function of $g(x)$ where $g(x)=\begin{cases}0 & if \quad 0<x\le\frac{1}{3}\\ 1 & if \quad \frac{1}{3}<x<\frac{2}{3}\\ 2 & if \quad x>\frac{2}{3}\end{cases}$.

26. Distribution function of a random variable X is given by $F(x)=\begin{cases}0, & x<0\\ x^2, & 0\le x\le 1\\ 1, & x>1\end{cases}$.

    Find the probability density function of X. Also determine (a) $P(X\le 0.5)$ (b) $P(0.5\le x\le 0.8)$ and (c) $P(X>0.9)$.

27. Let the joint probability distribution of (X, Y) be

(x, y)	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)	(2, 0)	(2, 1)	(2,2)
$f(x, y)$	1/15	3/15	2/15	2/15	2/15	1/15	1/15	1/15	2/15

    Find (a) the marginal distributions of X and Y (b) $P(Y \mid X=0)$ (c) $P(X\le 1 \mid Y\ge 1)$.

28. If X is a random variable having probability density function $f(x)=\begin{cases}\frac{x+1}{2}, & |x|\le 1\\ 0, & |x|>1\end{cases}$, find variance of X.

29. Derive Cauchy — Schwartz inequality.

30. Find the moment generating function of a random variable X with probability mass function $P(X = x) = {}^{n}C_{x}p^{x}q^{n-x}$, $x = 0,1,2,\ldots,n$. $p + q = 1$. Also derive $E(X)$ and $E(X^2)$ using m.g.f.

31. Explain bivariate moment generating function and write its properties.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) For n events $A_1, A_2 \ldots, A_n$ prove that $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$.

    (b) It is found that in 70% of cases Dr. Raj diagnosed a disease X correctly. The chance that a patient will die by his treatment after correct diagnosis is 20% and the chance of death by wrong diagnosis is 70%. A patient of Dr. Raj, who had disease X, died. What is the probability that his disease was diagnosed correctly?

33. Derive the characteristic function of a random variable X with probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{0}\right)^2}$, $-\infty < x < \infty$, $-\infty < \mu < x$, $\sigma > 0$.

34. Let the joint probability density function of a bivariate random variable (X, Y) be $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & otherwise \end{cases}$. Find the conditional mean and conditional variance of $X \mid Y = y$.

35. Consider a bivariate random variable (X, Y) with joint probability density function $f(x) = \begin{cases} \alpha^{-2} e^{-(x+y)/\alpha}, & x, y > 0, \alpha > 0 \\ 0, & otherwise \end{cases}$. (a) Test whether X and Y are independent or not (b) Find the distribution of $\frac{1}{2}(X - Y)$.

(2 × 15 = 30 Marks)
