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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1331.4 — MATHEMATICS III – FOURIER SERIES, NUMERICAL METHODS AND ODE

(2019 Admission – Regular)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries 1 mark.

1. Determine the period of the function $\cos^2 x$.

2. How to identify a given function is odd or even?

3. Write down the Fourier cosine series.

4. Define general solution of a differential equation.

5. Find the degree of the differential equation $\left[1+\left(\frac{dy^2}{dx}\right)\right]^4 = \left(\frac{d^2y}{dx^2}\right)^3$.

6. Write short note about singular solution.

7. Check the equation $x^3\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$ is homogeneous linear equation or not.

8. Find the integrating factor of $\frac{dy}{dx} + y\tan x = \cos^3 x$.

P.T.O.

9. Write the Newton — Raphson iteration formula.

10. Write down the expression for trapezoidal rule?

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Write down at least three Dirichlet's condition for a Fourier series.

12. Find the Fourier coefficient b_n of the function $f(x) = \begin{cases} -x & if \quad -\pi < x < 0 \\ x & if \quad 0 < x < \pi \end{cases}$.

13. Let n be any positive integer. Prove that sin nx is a periodic function with period $2\pi/n$.

14. Find the Fourier transform of $f(x) = \begin{cases} 1 & for \quad |x| < a \\ 0 & for \quad |x| > a \end{cases}$

15. Find the differential equation associated with the primitive $y = \cos(x+3)$.

16. Solve $(x+1)\dfrac{dy}{dx} = x(x^2+1)$

17. Solve the equation $\dfrac{dy}{dx} + \dfrac{1}{x}y = x^3 - 3$.

18. Test the differential equation $(e^x + 1)\cos x dx + e^x \sin x dy = 0$ is exact or not?

19. Solve the differential equation $xp^2 + x = 2yp$.

20. Explain the term complementary function of linear differential equation.

21. Solve $(D^2 + 3D + 2)y = 2x$.

22. Find the differential equation of all circles passing through the origin and having their centers on the x axis.

23. Define an algebraic expression with suitable example.

24. Write short note about Gaussian elimination method for system of linear equations.

25. Write short note about trapezoidal rule for numerical integration.

26. Write short note about Simpsons rule for numerical integration.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

27. If $f(x) = \begin{cases} -1 & in \quad -\pi < x < 0 \\ 1 & in \quad 0 < x < \pi \end{cases}$ with period 2π, find the Fourier series for *f(x)*.

28. Find the Fourier series of the function $f(x) = x$ *for* $0 < x < 2\pi$

29. Represent the function $f(t) = \begin{cases} t & if \quad 0 < t \leq \frac{\pi}{2} \\ \frac{\pi}{2} & if \quad \frac{\pi}{2} < x \leq \pi \end{cases}$ by a Fourier sine series.

30. Obtain the complex form of the Fourier series of the function $f(x) = \begin{cases} 0 & if \quad -\pi < x \leq 0 \\ 1 & if \quad 0 \leq x < \pi \end{cases}$.

31. Solve $\frac{dy}{dx} = \frac{y}{x} + x \sin \frac{y}{x}$.

32. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$.

33. Solve $(D^2 + 5D + 4)y = 3 - 2x$.

34. Solve the Differential equation $\frac{d^2x}{dt^2} + \frac{g}{l}x = \frac{g}{l}L$ where g, l, L are constants subject to the conditions. $x = a, \frac{dx}{dt} = 0$ at $t = 0$.

35. Use Taylofs series method, to solve the equation $\frac{dy}{dx} = -xy$, $y(0) = 1$.

36. Use the Newton – Raphson method, to find the real root of $x = e^{-x}$, correct to three decimal places with $x_0 = 1$.

37. Apply the trapezoidal rule to obtain a value of the integral $\int_{4}^{5} f(x)dx$ given

x:	4.0	4.2	4.4	4.6	4.8	5.0
f(x):	1.386	1.435	1.482	1.526	1.569	1.609

38. Evaluate $\int_{2.2}^{2.8} \frac{x}{1+3x} dx$ using Simpson's 1/3 rule.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Develop the Fourier series off $f(x) = \begin{cases} \frac{1}{2} + x & if \quad -\frac{1}{2} < x < 0 \\ \frac{1}{2} - x & if \quad 0 < x < \frac{1}{2} \end{cases}$

40. Find the Fourier series for $f(x) = x^2$ in $-\pi < x \leq \pi$ and reduce that

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

41. Solve $\frac{dy}{dx} = \frac{x + 9y - 20}{6x + 2y - 10}$.

42. Solve completely $(D^2 - 4D + 4)y = e^{2x} + x^2 + \cos 2x$.

43. Apply Runge – Kutta method to find an approximate value of y when $x = 0.02$, given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$.

44. Use Gauss – Seidel method solve the system of equations $3x + y + z = 1$; $x + 3y - z = 11$; $x - 2y + 4y = 21$.

(2 × 15 = 30 Marks)