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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Mathematics

Core Course – III

MM 1441 — ELEMENTARY NUMBER THEORY AND CALCULUS – II

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** *questions are compulsory. They carry* **1** *mark* **each**.

1. Determine the number of incongruent solutions of the linear congruence $12x \equiv 18 \pmod{15}$.

2. State true or false. "If $a^2 \equiv b^2 \pmod{m}$, then $a \equiv b \pmod{m}$".

3. Determine whether 327723 is divisible by 6.

4. State Fermat's Little Theorem.

5. If $r(u, v) = (1-u)i + [(1-u)\cos v]j + [(1-u)\sin v]k$, then find $\frac{\partial r}{\partial u}$ and $\frac{\partial r}{\partial v}$.

6. Find a parametric representation of the surface $z = 4 - x^2 - y^2$.

7. Evaluate $\int_0^1 x^2 y\, dy$.

8. Let T be the transformation from the uv-plane to the xy-plane defined by the equations $x = \frac{1}{4}(u+v)$, $y = \frac{1}{2}(u-v)$. *Find* $T(1, 3)$.

9. Determine whether the vector field $F(x, y) = (y + x)i + (y - x)j$ is conservative on some open set.

10. Write a formula for a general inverse-square field $F(r)$ in terms of the radius vector r.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Prove that if $a \equiv b(\text{mod } m)$, then $a^n \equiv b^n(\text{mod } m)$ for any positive integer n.

12. Find the remainder when 1! + 2! + ... + 100! is divided by 15.

13. Using inverse, find the incongruent solutions of the linear congruence $5x \equiv 3(\text{mod } 6)$.

14. Find the least residues x such that $x^2 \equiv 1(\text{mod } 8)$.

15. Evaluate $\int_1^3 \int_2^4 (40 - 2xy)\, dy\, dx$.

16. Find the volume of the solid enclosed by the surface $z = x/y$ and the rectangle $0 \le x \le 4, 1 \le y \le e^2$ in the xy-plane.

17. Find the Jacobian $\frac{\partial(x, y)}{\partial(r, \theta)}$ of the transformation $x = r\cos\theta$, $y = r\sin\theta$.

18. Use a double integral to find the area of the region R enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

19. Sketch the vector field $F(x, y) = -yi + xj$.

20. Find $divF$ and curl F for the vector field $F(x, y, z) = x^2 i + y^2 j + z^2 k$.

21. Evaluate the line integral $\int_C (xy + z^3) ds$ $(1,0,0)$ to $(-1, 0, \pi)$ along the helix C that is represented by the parametric equations $x = \cos t, y = \sin t, z = t, 0 \leq t \leq \pi$.

22. Let $F(x, y) = (ye^{xy} - 1)i + xe^{xy} j$. Find a potential function for F.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions among the questions **23** to **31.** These questions carry **4** marks **each.**

23. Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares.

24. Using the Pollard rho method with $x_0 = 2$ and $f(x) = x^2 + 1$, find the canonical decomposition of 3893.

25. Find the least positive integer that leaves the remainder 1 when divided by 3, 2 when divided by 4, and 3 when divided by 5.

26. Show that the congruence relation is an equivalence relation.

27. Use a polar double integral to find the area enclosed by the three-petaled rose $r = \sin 3\theta$.

28. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

29. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes z = 1 and $x + z = 5$.

30. Find the work done by the force field $F(x,y) = (e^x - y^3)i + (\cos y + x^3)j$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction using Greens theorem.

31. Suppose that a semi-circular wire has the equation $y = \sqrt{25 - x^2}$ and that its mass density is $\delta(x,y) = 15 - y$. Find the mass of the wire.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions among the questions **32** to **35**. These questions carry **15** marks **each.**

32. (a) State and prove Euler's theorem.

    (b) Find the remainder when 18! is divided by 23.

33. (a) Find the volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$.

    (b) Use cylindrical coordinates to evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2 \, dz \, dy \, dx$.

34. (a) Evaluate $\iint_R e^{xy} \, dA$, where R is the region enclosed by the lines $y = \frac{1}{2}x$ and $y = x$ and the hyperbolas $y = \frac{1}{x}$ and $y = \frac{2}{x}$.

    (b) Evaluate the surface integral $\iint_\sigma x^2 \, dS$ over the sphere $x^2 + y^2 + z^2 = 1$.

35. (a) State Divergence Theorem and verify it for the field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

    (b) Suppose that a curved lamina σ with constant density $\delta(x,y,z) = \delta_0$ is the portion of the paraboloid $z = x^2 + y^2$ below the plane z = 1. Find the mass of the lamina.

(2 × 15 = 30 Marks)
