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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programme Under CBCSS

Statistics

Core Course

ST 1441 – PROBABILITY AND DISTRIBUTION II

(2018 Admission)

Time : 3 Hours Max. Marks : 80

Instructions: Scientific Calculators and Mathematical/statistical tables are permitted to use.

SECTION – A

(Answer **all** questions. Each question carries **1** mark)

1. Find mean and variance of a Bernoulli distribution with parameter p?

2. If X – B(10,0.4) find P(Y=3) where Y = 10 – X

3. If X is a Poisson random variable with P(X = 0) – e^{-2}, find mean and variance of the distribution.

4. Two percent of tools produced in a certain manufacturing process turn out to be defective Using Poisson approximation, find the probability that in a sample of 200 units, exactly 2 units are defective.

5. An unbiased coin is tossed until a head is obtained, If X is the number of tosses required, find the probability mass function of X.

6. Write down the probability mass function of a negative binomial distribution.

7. If the moment generating function of X is $\frac{3}{4-e^t}$, identify the distribution.

8. If X and Y are two independent standard normal random variables, what is the distribution of 2X + 3Y?

9. For a Uniform random variable over (-2,2) find $P(|x|>1)$.

10. For a Cauchy distribution with probability density function $f(x)=\frac{1}{\pi 1+x^2} -\infty < x < \infty$ find the distribution function.

(10 × 1 = 10 Marks)

SECTION – B

(Answer **any eight** questions. Each question carries **2** mark)

11. Suppose X is a discrete uniform random variable with values 1,2....n Find the variance of the distribution.

12. Obtain moment generating function of a Binomial random variable with n=7 and p = 0.4 and hence find its Cumulant generating function.

13. If X is a Poisson random variable such that P(X = l)= 2 P(X = 2) find Mean and Variance of X.

14. If X and Y are independent Poisson random variables with parameters λ_1 and λ_2 respectively, find P(X=Y).

15. Find the mean of a geometric distribution with parameter p.

16. Define Multinomial distribution.

17. Define bivariate normal distribution.

18. Find the characteristic function of an exponential random variable with parameters θ

19. Given that X is normally distributed with mean 10 and P(X > 12) = 0.1587 find its standard deviation.

20. Find the moment generating function of a standard normal random variable.

21. Write down the characteristic function of a standard Cauchy distribution and hence find the distribution of $\frac{1}{n}\sum_{1-1}^{n} x_i$ where $X_1, X_2 ... X_n$ are independent and identically distributed standard Cauchy random variables.

22. Write down the probability density function of random variable that follows Beta distribution of second kind. What is the transformation required in order to transform a random variable of Beta distribution of second kind to Beta distribution of first kind.

(8 × 2 = 16 Marks)

SECTION – C

(Answer **any six** questions. Each question carries **4** mark)

23. Obtain the moment generating function of a Binomial random variable and using this function derive the additive property of Binomial variables.

24. For a Poisson distribution with parameter λ find the mode of the distribution. When do you say the Poisson distribution is a bimodal distribution?

25. If X – P(λ_1) and X – P(λ_2) are two independent Poisson random variables then show that the conditional distribution of X given X+Y is Binomial.

26. If X and Y are two independent geometric random variables with same probability of success P show that $P(X = Y) = \frac{P}{2-p}$.

27. Define Hyper geometric distribution. Find the mean of the distribution.

28. For a Uniform random variable with probability density function $f(x) = \frac{1}{20}, -0 \le x \le 0$ derive coefficient of skewness and Kurtosis of the distribution.

29. For a normal distribution with mean μ and variance σ^2, find the Quartile deviation.

30. For a normal distribution 7% items are under 35 and 89% are under 63.Find the mean and variance of the distribution.

31. Suppose (X. Y) – BN($\mu_1, \mu_2, \sigma_1^2 \sigma_2^2, \rho$) is a bivariate normal random vector, then establish the condition $\rho = 0$ for independence of X and Y.

(6 × 4 = 24 Marks)

SECTION – D

(Answer **any two** questions. Each question carries **15** mark)

32. (i) The probability that a man hitting the target is 1/3.How many times must he fire so that the probability of hitting the target at least once is more than 90%?

(ii) The probability of getting no misprint in a page of a book is e^{-4}. What is the probability that a page contains more than 2 misprints?

(iii) Let X and Y be two independent normal random variables with equal means and standard deviations 2 and 3 respectively and let P(4X + 2Y ≤ 3) = P(2X – Y ≥ – 4), determine the common mean of X and Y.

33. Derive the recurrence relation between central moments of a Normal distribution with mean μ and variance σ^2. Hence find coefficient of skewness and Kurtosis of the distribution.

34. (i) If X is a gamma random variable with parameter (α, β),find its moment generating function and hence find mean of the distribution.

(ii) If X is a Beta distribution of first kind with parameters p and q find the mean and Harmonic mean of X.

35. (i) Define log normal distribution and find its r^{th} moment.

(ii) Let $X_i - \exp(0_i), i = 1,2...n$ be n independent exponential random variables then find distribution function of the random variable Y= min(X_1,X_2....X_n).

(2 × 15 = 30 Marks)