(Pages : 6) **J – 3007**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, June 2020

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1431.4 : MATHEMATICS IV – LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the norm of the vector $\begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$.

2. Check whether the vectors $u = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$ are orthogonal.

3. Given $u = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$, find $2u - 3v - w$.

P.T.O.

4. Are the matrices A and B row equivalent?

$$A=\begin{bmatrix} 1 & -2 & -1 & 3 & 0 \\ -2 & 4 & 5 & -5 & 3 \\ 3 & -6 & -6 & 8 & 2 \end{bmatrix}, \; B=\begin{bmatrix} 1 & -2 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}.$$

5. Find the inverse of $A=\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$.

6. Check whether the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ is singular.

7. Find the rank of the matrix $\begin{bmatrix} 5 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

8. Define dilation from $\mathcal{R}^2$ to $\mathcal{R}^2$.

9. What is the order of the matrix of the transformation $T : \mathcal{R}^7 \to \mathcal{R}^4$?

10. Write the standard matrix corresponding to the transformation of reflection through the $x_2 = -x_1$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. They carry **2** marks each.

11. Verify Cauchy Schwarz inequality for the vectors $x=(1, 1)$, $y=(5, -1)$

12. Is the third quadrant a vector subspace of $\mathcal{R}^2$? Justify your answer.

13. Check whether the vectors $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ are linearly independent in $\mathcal{R}^3$.

14. Define dimension of a vector space.

15. If $A=\begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix}$, show that $A^2+2A-5I=0$.

16. Find the eigen values of the matrix $\begin{bmatrix} 7 & 4 \\ -3 & 1 \end{bmatrix}$.

17. If 3 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & 1 \\ 1 & -1 & 3 \end{bmatrix}$, find all the eigen values of A^{-1} without using characteristic equation.

18. By reducing to echelon form, find the rank of the matrix $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$.

19. Check whether $T : \mathcal{R}^2 \to \mathcal{R}^3$ given by $T[(x_1, x_2)] = (x_1 - 2x_2, 0, 3x_1 - 2x_2)$ is a linear transformation..

20. Find the standard matrix of the linear transformation $T : \mathcal{R}^2 \to \mathcal{R}^2$ which is rotation through $\frac{\pi}{2}$ radians.

21. Find the B–co-ordinate vector of x where $\{b_1, b_2\}$, $b_1=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $b_2=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x=\begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

22. Find a basis for the null space and column space of the matrix $\begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 23 to 31. They carry **4** marks each.

23. Solve the system of equations by Cramer's rule :

$$x + y - z = 9$$
$$y + 6z = -6$$
$$-2x + 4y - 6z = 4.$$

24. For what value of λ and μ do the system of equations :

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + \lambda z = \mu$$

has (a) no solution (b) unique solution (c) infinite solution.

25. Find the eigen values and eigen vectors of $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

26. Show that $\mathcal{B} = \{b_1, b_2, b_3\}$ form a basis of $\mathcal{R}^3$ where $b_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}$,

$b_3 = \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$.

27. Determine whether the transformation, $T : \mathcal{R}^3 \to \mathcal{R}^2$ given by $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$ is

(a) one-one

(b) onto.

28. Diagonalize the matrix $\begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$.

29. Let $\mathcal{B}=\{b_1, b_2\}$, $\mathcal{C}=\{c_1, c_2\}$ be bases of $\mathcal{R}^2$ where $b_1=\begin{bmatrix} -6 \\ -1 \end{bmatrix}$, $b_2=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $c_1=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $c_2=\begin{bmatrix} 6 \\ -2 \end{bmatrix}$. Find the change of co-ordinate matrix from B to C and the change of co-ordinate matrix from C to B.

30. Find the B-matrix of the transformation $T : \mathcal{R}^2 \to \mathcal{R}^2$ given by $T(x)=Ax$, where $A=\begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$, $b_1=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b_2=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathcal{B}=\{b_1, b_2\}$.

31. Prove that the vectors $u=\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $v=\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $w=\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ are linearly dependent and find the linear dependence relation between them.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. They carry **15** marks each.

32. Find an orthonormal basis for the subspace spanned by the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ of $\mathcal{R}^3$, using Gram Schmidt process.

33. Classify the quadratic form $2x_1^2 + 10x_1x_2 + 2x_2^2$ and make a change of variable $x = Py$ that transforms the quadratic form into one with no cross product term. Write P and the new quadratic form.

34. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$.

35. Define $T : \mathcal{R}^2 \to \mathcal{R}^2$ given by $T(x) = Ax$ where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. Find a basis of $\mathcal{B}$ of $\mathcal{R}^2$. with the property that $[T]_{\mathcal{B}}$ is diagonal

(2 × 15 = 30 Marks)
