(Pages : 4) **H – 2106**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree Programme Under CBCSS

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. What is the first derivative of $\ln(a^x + a^{-x})$?

2. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 100$.

3. Derivative of $f(x) = \ln\left(\frac{x}{1+x^2}\right)$ with respect to x is ————.

4. Find all values of c in the interval $[3, 5]$ that satisfy the conclusion of the Rolle's theorem for the function $f(x) = x^2 - 8x + 15$.

5. $\left(\sum_{n=1}^{\infty}\frac{1}{n}\right) - 1 =$ ————.

6. For what values of x, the power series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ converges.

P.T.O.

7. If R is the radius of convergence of a power series then the radius of convergence of its derived series is ————————.

8. Define absolute convergence.

9. $\int \ln x \, dx =$ ————————.

10. $\frac{d}{dx}\left[\int_1^x t^3 dt\right] =$ ————————.

SECTION – II

Answer any **eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Find the position and nature of the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 2$.

12. Using Leibnitz's theorem find the second derivative of $\cos x \sin 2x$.

13. Find the intervals on which the function $f(x) = x^2 - 5x + 6$ is increasing and decreasing.

14. Let $f(x) = \begin{cases} 3x^2 & x \le 1 \\ ax + b & x > 1 \end{cases}$. Find the values of a and b so that f will be differentiable at $x = 1$.

15. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{(n!)^2}$ converges.

16. Discuss the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ to ∞.

17. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.

18. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

19. Evaluate $\int_2^5 (2x - 5)(x - 3)^4 \, dx$.

20. At what point or points in the interval $[0, 1]$ does the function $f(x) = 3x^2 - 1$ assume its average values?

21. If the power series $P(x)$ converges for $x \in (a, b)$. Show that $\frac{d}{dx}P(x)$ and $\int P(x)dx$ converge for all $x \in (a, b)$.

22. Evaluate $\int_0^{\infty} \frac{xdx}{(x^2 + a^2)^2}$.

SECTION – III

Answer any **six** questions from among the questions 23 to 31. These questions carry 4 marks each.

23. Let $f(x) = x^{2/3}$, $a = -1$, $b = 8$. Show that there is no point c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. Explain why the result does not contradict the Mean-Value theorem.

24. Find the radius of curvature at any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. What is the curvature when $b = a$?

25. Test whether the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ is convergent or not.

26. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

27. Sum the series $S(\theta) = 1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

28. Find the values of x for which the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$ converges.

29. Using integration by parts, find a relationship between In and I_n, where $I_n = \int_0^1 (1 - x^3)^n dx$ and n is any positive integer. Hence evaluate $I_2 = \int_0^1 (1 - x^3)^2 dx$

30. Find the total are between the curve $y = 1 - x^2$ and the x-axis over the interval [0, 2].

31. Evaluate $\int_0^1 \tan^{-1} x \, dx$.

SECTION – IV

Answer any **two** questions from among the questions 32 to 35. **These** questions carry **15** marks each.

32. (a) Use the Mean-value theorem to prove that $\frac{x}{1+x^2} < \tan^{-1} x < x (x < 0)$.

(b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

33. (a) Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

(b) Find the Maclaurin series for $x \sin x$.

34. (a) Find the area of the surface generated by resolving the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ about the x–axis.

(b) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}, y = 2$ and $x = 0$ is resolved about the y–axis.

35. (a) Find the Taylor series generated by $f(x) = \frac{1}{x-1}$ at $x = 2$. Where does the series converge to $f(x)$?

(b) Find the area of the surface generated by resolving the curve $y = x^3$, $0 \leq x \leq \frac{1}{2}$ about the x–axis.