Mathematics & Statistics

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G – 2477

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Mathematics

Complementary Course

MM 1231.4 MATHEMATICS II – ADVANCED DIFFERENTIAL AND INTEGRAL CALCULUS

(FOR STATISTICS)

(2018 ADMISSION)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **all** questions. Each carries 1 mark :

1. Check whether the differential $3\cos x dy + \sin y dx$ is exact.

2. Give the Taylor series expansion of a function $f(x, y)$ of two variables in x and y about a point (x_1, y_1)

3. Write the sufficient condition for which a point (x, y) to be a stationary point of the function $f(x, y)$.

4. Evaluate $\int_1^2\int_1^3 x y^2 dx\, dy$.

5. Write a Jacobian of the transformation $V = f(x, y)$ $v = g(x, y)$ with respect to x and y.

P.T.O.

6. The value of $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1} xy^2 z\,dx\,dy\,dz$ is .

7. Write the value of the constant $\Gamma(1/2)$.

8. Write the relation between a Beta and Gamma integrals.

9. The value of $B(5, 3)$ is .

10. The value of $\Gamma(5)$ is. **(10 × 1 = 10 Marks)**

PART – B

Answer **any eight** questions. Each question carries 2 marks.

11. If $f(t) = t^2$ $g(t) = t$, find the rate of $\phi(x, y) = 1 + xy$ with respect to 6.

12. Find the total derivative of $f(x, y) = x^2 + x^4 y$ with respect to x given that $y = x^2$.

13. Check whether $2xyzdx + x^2zdz + x^2ydz$ is exact.

14. Find the critical points of the functions $f(x, y) = x^3y^2(1 - x - y)$.

15. Evaluate $\iint xydxdy$ over the region in the positive equation for which $\frac{x}{a} + \frac{y}{b} \leq 1$.

16. Find the moment of inertia of a uniform rectangular lamina of mass M with sides a and b about one of sides of length a.

17. Evaluate $\iint_R xy\,dy\,dx$ where R is the region bounded by the x-axis, the ordinate $x = za$ and the parabola $x^2 = 4ay$, $a > 0$.

18. Prove that $\beta(m+1, n) + \beta(m, n+1) = \beta(m, n)$.

19. Establish the symmetry rotation between the variables of a beta function.

20. Find the value of $\beta(1, 1/2)$.

21. Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\pi}}{4} \frac{\Gamma(1/4)}{\Gamma(3/4)}$.

22. Prove that $\Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5......(2n-1)}{2^n}\sqrt{\pi}$ where n is a positive integer.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries 4 marks.

23. For the functions $f(x, y) = 2x^3y^2 + a$ verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

24. Find the points on the surface $z^2 = xy + 1$ that are nearest to the origin.

25. A rectangular box open at the top is to have a volume of 32 cubic units. Find the dimensions of the box requiring the least material for this constructions.

26. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

27. Evaluate $I = \iiint_R xydxdydz$ where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

28. If R is the region bounded by the planes $x = 0, b = 0, z = a$ and the cylinder $x^2 + y^2 = 1$, evaluate the integral $\iiint_R xy\,z\,dx\,dy\,by$ changing it into cylindrical coordinates.

29. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

30. Prove the duplication formula $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n)$.

31. Express the integral $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of the beta functions and hence evaluate $\int_0^1 x^7(1-x^4)^3\,dx$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries 15 marks.

32. If $x=e^u\cos\theta, y=e^u\sin\theta$ then show that the relation $\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial\theta^2}=(x^2+y^2)\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right)$ where $f(x, y)=\phi(u, \theta)$.

33. (a) Evaluate $\iint_R (x+y)^2\,dxdy$ where R is the region bounded by the circle $x^2+y^2=a^2$.

(b) Evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration.

34. (a) Using tripple integrals, find the volume of the cube $\{(x, u, z); 0\le xyz\le a^3\}$ $a>0$.

(b) Find the volume of the half sphere $\{(x, y, z) \mid 0\le x^2+y^2+z^2\le a^2, z>0\}$.

35. (a) Prove that $B(m, n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$.

(b) Prove that $\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{{}_2 2n-1}\Gamma(2n)$.

(2 × 15 = 30 Marks)