(Pages : 6) **G – 2468**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019.

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **ten** questions are compulsory and each carries 1 mark.

1. Construct a truth table for the compound statement $\sim(p \wedge q) \Leftrightarrow [(\sim p) \vee (\sim q)]$.

2. Write the negation statement of "If (a_n) is monotone and bounded, then (a_n) is convergent.

3. If the function $g(n, m) = n^2 + n + m$, where n and m are positive integers. Then find $g(16, 17)$.

4. Let f be the function given by $f(x) = 4x + 7$. Use the contrapositive implication to prove the statement. If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

5. Given an example of a relation which is reflexive but not symmetric and not transitive.

6. Find the slope of $x = \sec t, y = \tan t$ at $t \pi/3$.

7. Find the arc length of the spiral $r = e^{\theta}, 0 \leq \theta \leq \overline{n}$.

8. Find an equation for the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$.

9. Find the norm of $v = -3i + 2j + k$.

10. Find the vector orthogonal to both of the vectors $\overline{u} = (2, -1, 3)$ and $\overline{v} = (-7, 2, -1)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions 11 to 22. Each question carries **2** marks.

11. Write the four different types of negation statement of "$\forall \xi > 0 \exists N \to \forall n$, if $n \geq N$, then $\forall x$ in S, $\| fn(x) - f(x) \| < \xi$".

12. If x in rational and y in irrational, than prove that xy is irrational.

13. Let $S = \{x \in R, x > 0\}$. For each $x \in S$, let $A_x\left(-\frac{1}{X}, \frac{1}{X}\right)$. Find $\bigcap_{x \in S} Ax$.

14. Define a relation R on N × N by $(a, b)R\,(c, d)$ iff $a^d = c^d$.

    (a) Find an equivalence class with exactly two elements.

    (b) Find an equivalence class with exactly tour elements.

15. Let $f: A \to B$ and $g: B \to C$ be two injective functions. Show that the composition $g \circ f: A \to C$ is injective.

16. Find all values of t at which the parametric curve. $x = 2\sin t, y = 4\cos t$ $(0 \leq t \leq 2\pi)$ has horizontal tangent line.

17. Find the polar coordinates of the point P whose rectangular coordinates are $\left(-2, -2\sqrt{3}\right)$.

18. Sketch the graph $r = \cos 2\theta$. Also verify it is symmetric or not.

19. Find the area of the region inside the cardioid $r = 2 + 2ws\theta$ and outside the circle $r = 3$.

20. Describe the surface whose equation is $2x^2 + 2y^2 + 2z^2 - 2x - 3y + 5z - 2 = 0$.

21. Find the angle between a diagonal of a cube and one of its edges.

22. Let u and v be non-zero vectors in 3 – pace, and let θ be the angle between these vectors when they are positioned so their initial points coincide. Prove that $\|u \times v\| = \|u\| \|v\| \sin\theta$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions from questions 23 to 31. Each question carry **4** marks.

23. Define a new sentential connective ∇, called nor by the following truth table.

p	q	p∇q
T	T	F
T	F	F
F	T	F
F	F	F

(a) Use a truth table to show that p∇p is logically equivalent to ~ p.

(b) Complete a truth table for $(p\nabla p) \vee (q\nabla q)$.

24. Find $\bigcup_{B\in \mathscr{B}} B$ and $\bigcap_{B\in \mathscr{B}} B$ or each collection $\mathscr{B}$.

(a) $\mathscr{B} = \left\{\left[1+1+\frac{1}{n}\right] : n \in N\right\}$,

(b) $\mathscr{B} = \left\{\left[1+1+\frac{1}{n}\right] : n \in N\right\}$,

(c) $\mathscr{B} = \{[2, x], x \in R \text{ and } n > 2\}$

(d) $\mathscr{B} = \{[0, 3], [1, 5], [2, 4]\}$.

25. Determine which of the three properties (reflexive, symmetric and transitive) apply to each relation.

(a) Let S be the set of people in the school. Define R on S xRY iff "n likes y"

(b) Let R be the relation on R given by xRY iff $|x - y| \leq 2$.

26. Sketch the curve by eliminating the parameter and indicates the direction of increasing t

(a) $x = 3t - 4$, $y = 6t + 2$

(b) $x = \sec t, y = \tan t \left(\pi, t \leq 3\frac{\pi}{2}\right)$.

27. Describe the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$.

28. Find the total arc length of the cardioid $r = 1 + \cos\theta$.

29. Determine whether $u = \langle 4, 1, 6\rangle$ and $v = \langle -3, 0, 2\rangle$ make an a cute angle, an obtuse angle or orthogonal.

30. Find the parametric equations of the line that satisfies the stated conditions :

(a) The line that is tangent to the circle $x^2 + y^2 = 25$ at the point (3, –4)

(b) The line through the origin that in parallel to the line given by $x = t, y = -1 + 4, z = 2$.

31. Sketch the graph of hyperboloid of two sheets $z^2 - x^2 \frac{y^2}{4} = 1$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** from questions 32 to 35. Each question carry **15** marks.

32. Prove the following :

(a) There exists an integer n such that $n^2 + 3\frac{n}{2} = 1$. Is this integer unique?

(b) For every real number $x > 5$, there exists a real number $y < 0$ such that $x = \frac{5y}{y+3}$.

(c) $\log_2 3$ is irrational.

(d) For every positive integer $n^2 + 4n + 8$ is even.

33. (a) Define a relation R on the set of all integers Z by xRY iff $x = y = 2k$ for some integer K. Verify that R is an equivalence relation and describe the equivalence class E_5. How many distinct equivalence classes are there?

(b) Let $f : A \to B$ and $g : B \to C$ by bijective functions show that the composition $g \circ f : A \to C$ is bijective and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

34. (a) Find the slope of the tangent line to the circle $r = 4\cos\theta$ at the point where $\theta = \frac{\pi}{4}$.

(b) Find the points on the cardioid $r = 1 - \cos\theta$ at which there is a horizontal tangent line, a vertical tangent line or a singular point.

35. (a) Find the equation of the plane through the points $P_1(1, 2, -1), P_2(2, 3, 1)$ and $P_3(3, -1, 2)$.

(b) Determine whether the planes $3x - 4y + 5z = 0$ and $-6x + 8y - 10z - 4 = 0$ are parallel.

(c) Let $L_1 : x = 1 + 4t, y = 5 - 4t, z = -1 + 5t, L_2 : x = 2 + 8t, \ y = 4 - 3t, z = 5 + t$ be two lines. Are the lines parallel?

(2 × 15 = 30 Marks)