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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2018
First Degree Programme under CBCSS
Complementary Course I for Statistics
MM 1131.4 : MATHEMATICS – I – Basic Calculus for Statistics
(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are **compulsory**. **Each** question carries **1** mark.

1. Find the stationary points of $f(x) = x^3 - 3x^2 + 3x$.
2. State Mean Value theorem.
3. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3xy$.
4. Find the sum of the geometric series $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$
5. Write the Maclaurin series for $\tan^{-1} x$.
6. Show that $\sum_{n=1}^{N} n^3 = \frac{1}{4} N^2(N+1)^2$.
7. Sum the integers between 100 and 500 inclusive.
8. Evaluate the integral $\int x \ln x \, dx$.
9. Evaluate the integral $\int_0^{\infty} e^{-x} \, dx$.
10. Find the mean value of the function $f(x) = x^2$ between the limits $x = 0$ and $x = 3$.

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SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. **Each** question carries **2** marks.

11. Find the positions and natures of the stationary points of the function $f(x) = x^3 - 3x + 1$.

12. Use Leibnitz' theorem to find the fourth derivative of $(2x^3 + 3x^2 + x + 2)\, e^{2x}$.

13. What semi-quantitative result can be deducted by applying Rolle's theorem to the function $x^2 + 7x + 3$ with a and c chosen so that $f(a) = f(c) = 0$?

14. Find all numbers b that satisfy the conclusion of the Mean value theorem for the function $f(x) = 2x^2 - 3x + 1$ in the interval [0, 2].

15. Let $S = \sum_{n=1}^{\infty} \frac{1}{n^2}$. By grouping and rearranging the terms, show that

    $$S_o = \sum_{n \text{ odd}}^{\infty} \frac{1}{n^2} = \frac{3S}{4}.$$

16. Test whether the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1}$ is convergent or divergent.

17. Sum the series $\sum_{n=1}^{N} (n-1)\,(n+2)$.

18. Test the alternating series $\sum_{n=0}^{\infty} \frac{(-1)^n n}{1+n^2}$ for convergence.

19. Sum the series $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots$

20. Evaluate $\int e^x \cos x \, dx$.

21. Evaluate $\int e^{\sqrt{x}} dx$ by making a substitution and then integrating by parts.

22. Evaluate $\int_1^3 \frac{dx}{(x-1)^{2/3}}$.

SECTION – III

Answer **any six** questions from among the questions **23** to **31**. **Each** question carries **4** marks.

23. Show that the radius of curvature at the point (x, y) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has magnitude $(a^4y^2 + b^4x^2)^{3/2}/(a^4b^4)$.

24. Determine whether the series $\sum_{n=1}^{\infty}\frac{n^2+3n+4}{n^4+7n^3+6n-3}$ converges or diverges, if it is known that $\sum_{n=1}^{\infty} n^{-2}$ is a convergent series.

25. Determine the range of values of x for which the series $\sum_{n=0}^{\infty}(2x)^n$ converges.

26. Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for p > 1 and diverges for p < 1.

27. Find the Maclaurin series for $\ln\left(\frac{1+x}{1-x}\right)$.

28. Use integration by parts to find a relationship between I_n and I_{n-2} where $I_n = \int_0^{\pi/2} x^n \cos x\,dx$ and n is any nonnegative integer.

29. Find the length of the curve $y = (4 - x^{2/3})^{3/2}$ over the interval [0, 8].

30. Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$, $1 \le y \le 4$ about the y-axis.

31. The line segment $x = 1 - y$, $0 \le y \le 1$ is revolved about the y-axis to generate a cone. Find the area of its surface excluding its base.

SECTION – IV

Answer **any two** questions from among the questions **32** to **35. Each** question carries **15** marks.

32. a) Use the difference method to sum the series $\sum_{n=2}^{N}\frac{2n-1}{2n^2(n-1)^2}$.

    b) Expand f(x) = cos x as a Taylor series about $x = \frac{\pi}{3}$.

33. a) Starting from the Maclaurin series for cos x, show that

$$(\cos x)^{-2} = 1 + x^2 + \frac{2x^4}{3} + \ldots$$

b) Find the range of values of z for which the complex power series

$$P(z) = 1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \ldots \text{converges.}$$

34. a) Show that the value of the integral $I = \int_0^1 \frac{1}{(1+x^2+x^3)^{1/2}}dx$ lies between 0.810 and 0.882.

b) The equation in polar coordinates of an ellipse with semi-axes a and b is

$$\frac{1}{p^2} = \frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}.$$ Find the area of the ellipse.

35. a) The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

b) Find the area of the surface generated by revolving the curve

$y = 2\sqrt{x}, 1 \le x \le 2$, about the x-axis.
