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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1141 : Methods of Mathematics
(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All questions **are** compulsory. **Each** question carries **1** mark.

1. What is the local linear approximation of the function y = f(x) at $x = x_0$?
2. State L'Hôpital's rule.
3. Define mathematically the terms *concave up* and *concave down* for a real function.
4. State first derivative test for relative maximum of a function f(x).
5. For a particle in rectilinear motion, the velocity and position functions v(t) and s(t) are related by the equation ____________
6. Area between the X-axis and the graph of the function $f:[a,b]\to\mathbb{R}$ is
7. The average value of a continuous function f on an interval [a, b] is
8. The volume of the solid that is obtained when the region under the curve y = f(x) over the interval [a, b] is revolved about the x-axis is
9. If y = f(x) is a smooth curve on the interval [a, b], then the arc length L of this curve over [a, b] is ________________
10. State Newton's second law of motion.

SECTION – II

Answer **any 8** questions. **Each** question carries **2** marks.

11. The diameter of a polyurethane sphere is measured with percentage error within ± 0.4%. Estimate the percentage error in the calculated volume of the sphere.

12. Evaluate $\lim_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$.

13. Find all critical points of $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$.

14. Find horizontal and vertical tangents of the curve $y = 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}}$.

15. Verify Rolle's theorem for $f(x) = x^2 - 5x + 4$ in the interval (1, 4)

16. Evaluate $\lim_{x \to 0} (\sin x)^{\tan x}$.

17. Suppose that a particle moves with velocity $v(t) = \cos(\pi t)$ along a coordinate line. Assuming that the particle has coordinate $s = 4$ at time $t = 0$, find its position at time t.

18. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

19. Derive the formula for the volume of a sphere of radius r.

20. State Pappus Theorem.

21. Find the fluid pressure and force on the top of a flat circular plate of radius 2 m that is submerged horizontally in water at a depth of 6 m.

22. Evaluate $\int_1^{\infty} \frac{dx}{x^3}$.

SECTION – III

Answer **any six** questions. **Each** question carries **4** marks.

23. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/s. How fast is the area of the spill increasing when the radius of the spill is 60 ft ?

24. Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.

25. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

26. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval [1, 5] and determine where these values occur.

27. Verify Mean Value Theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in [0, 1/2].

28. Use cylindrical shells to find the volume of solid generated when the region R under $y = x^2$ over the interval [0, 2] is revolved about the line $y = -1$.

29. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

30. A 100 ft length of steel chain weighing 15 /b/ft is dangling from a pulley. How much work is required to wind the chain onto the pulley ?

31. A plate in the form of an isosceles triangle with base 10 ft and altitude 4 ft is submerged vertically in machine oil. Find the fluid force F against the plate surface if the oil has weight density $\rho = 30$ /b/ft^3.

SECTION – IV

Answer **any two** questions. **Each** question carries **15** marks.

32. Let $f(x) = x^3 - 3x^2 + 1$. Determine the intervals on which f is increasing, decreasing, concave up, and concave down. Locate all inflection points of f. Also draw a rough sketch of the graph of f. **15**

33. a) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is avaliable for the fence ? **7**

    b) State and prove mean value theorem. **8**

34. a) A golfer makes a successful chip shot to the green. Suppose that the path of the ball from the moment it is struck to the moment it hits the green is described by $y = 12.54x - 0.41x^2$ where x is the horizontal distance (in yards) from the point where the ball is struck, and y is the vertical distance (in yards) above the fairway. Find the distance the ball travels from the moment it is struck to the moment it hits the green. Assume that the fairway and green are at the same level. 5

b) Find the surface area of the solid generated by revolving the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x-axis. 5

c) Obtain the volume of the solid generated by revolving the curve $x^2(y - x^2) = 3$ between $x = 1$ and $x = 2$ about x-axis. 5

35. a) Derive the work energy relationship. 5

b) Find the center of gravity of the triangular lamina with vertices (0, 0), (0, 1) and (1, 0) and density $\delta = 3$. 5

c) Derive the formula for the circumference of a circle of radius r. 5
