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Reg. No. : ....

Name : ....

Second Semester B.C.A. Degree Examination, July 2015
(Career Related First Degree Programme under CBCSS)
(2013 Admission Onwards)
Group 2 (b) : Complementary Course
MM 1231.9 : MATHEMATICS – II

Time: 3 Hours Max. Marks: 80

SECTION – I

All the first **10** questions are **compulsory**. **Each** question carries **1** mark. Answer in **one** word to maximum of **two** sentences :

1. State DeMorgan's laws.
2. What is meant by tautology ?
3. When will you say that a function $f : A \to B$ is invertible ?
4. Define equivalence relation.
5. Give an example of a totally ordered set.
6. Give an example of a non-abelian group.
7. Find the number of edges in a complete graph on 6 vertices.
8. Check whether the set Z of integers with binary operation $*$ such that $x * y = x^y$ is a semigroup or not.
9. Give an example of a regular graph on five vertices.
10. Define bipartite graph.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. They carry **two** marks **each**.

11. What is meant by Boolean expression ? Give an example.

12. Draw the truth tables of conditional and biconditional statements.

13. Obtain disjunctive normal forms of :

    a) $p \wedge (p \rightarrow q)$

    b) $\neg (p \vee q) \Leftrightarrow p \wedge q$.

14. Let R be an equivalence relation on a set A. Prove that R induces a partition on A.

15. Is the following function one-one ?

    $f : N \rightarrow N$ where $f(n) = \begin{Bmatrix} 2n, & n \text{ is even} \\ n, & n \text{ is odd} \end{Bmatrix}$.

16. Let f (x) = x + 3, g (x) = x – 4, h (x) = 5x are functions from R to R, where R is the set of real numbers. Show that f o (goh) = (fog) oh.

17. Let Z be the set of integers and let T be the set of all even integers. Show that the semigroups (Z, t) and (T, t) are isomorphic.

18. Define Hamming distance and state its properties.

19. Prove that in a simple digraph sum of out degree of all the vertices is equal to the sum of in degree of all vertices and this sum is equal to the number of edges.

20. Construct a DFSA M that accept exactly the strings of x's and y's that have even number of y's.

21. Prove that in a group G, the identity element and inverse of an element is unique.

22. Draw the complete graph on 4 vertices and find its adjacency matrix.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. They carry **four** marks **each**.

23. Explain the resolution principle with a suitable example.

24. Test the validity of the argument.

    $p \rightarrow q$

    $\underline{q}$

    p

25. Let A = {1, 2, 3, 4} and R be the relation {(1, 2), (2, 3), (3, 4), (2, 1)}. Find the transitive closure by using Warshal's algorithm.

26. Let A be the set of non-zero integers and let $\approx$ be the relation on $A \times A$ defined by $(a, b) \approx (c, d)$ whenever ad = bc. Check whether $\approx$ is an equivalence relation.

27. Define bijection. Show that if f and g are bijections from X to Y and Y to Z respectively, then $gof : x \rightarrow z$ is also a bijection.

28. Define group isomorphism. Let G be the group of real numbers under addition and let G' be the group of positive real numbers under multiplication. Show that $f : G \rightarrow G'$ defined by $f(x) = e^x$ is an isomorphism.

29. Define ring and give an example.

30. Explain the depth-first search algorithm with a suitable example.

31. Sketch the graph of each function :

    a) $f(x) = \frac{1}{2}x - 1$

    b) $g(x) = \begin{Bmatrix} 0, & \text{if } x = 0 \\ 1/x, & \text{if } x \neq 0 \end{Bmatrix}$.

SECTION – IV

Answer **any two** questions from among the questions **32** to **35**. They carry **15** marks **each**.

32. a) Test the validity of the argument :

    If a man is a bachelor, he is unhappy.

    If a man is unhappy, he dies young.

    Bachelors die young.

    b) Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

    c) Verify the proposition $p \vee \neg(p \wedge q)$ is a tautology.

33. a) Explain Breadth-first search algorithm with a suitable example.

    b) Find the inverse of the following functions :

    i) $f(x) = 2x - 3$

    ii) $g(x) = \frac{2x-3}{5x-7}$.

34. a) State and prove the inclusion-exclusion theorem in set theory.

    b) Prove that a hamming code can correct all combinations of k or fewer errors if and only if the minimum distance between any two code is atleast $2k + 1$.

35. a) Explain the communication model and error correction in detail.

    b) Let G be a directed graph. Show that a vertex v is the root of a strongly-connected component of G if and only if lowlink [v] = v.